Finite Population Trust Game Replicators

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Abstract. Our previous work introduced the N player trust game and examined the dynamics of this game using replicator dynamics for an infinite population. In finite populations, quantization becomes a necessity that introduces discontinuity in the trajectory space, which can impact the dynamics of the game differently. In this paper, we present an analysis of replicator dynamics of the N player trust game in finite populations. The analysis reveals that, quantization indeed introduces fixed points in the interior of the 2-simplex that were not present in the infinite population analysis. However, there is no guarantee that these fixed points will continue to exist for any arbitrary population size; thus, they are clearly an artifact of quantization. In general, the evolutionary dynamics of the finite population are qualitatively similar to the infinite population; which suggests that for the proposed trust game, trusters will be extinct if the population contains an untrustworthy player. Therefore, trusting is an evolutionary unstable strategy.
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1 Introduction

Human interaction is a complex process. Despite being the focus of extensive investigation for decades, a number of questions remain without adequate answers. Social dilemmas are particularly interesting. Social dilemmas arise whenever short-term, individual interests must be weighed against long-term, collective interests.

Game theory is a useful vehicle for studying social dilemmas. Players compete against each other using strategies to make decisions. They receive payoffs (rewards) for the decisions they make and their competitors make. Good strategies return high rewards. Theories can be postulated to explain how strategies might evolve over time and computer models can be constructed to generate empirical evidence to support or disprove these theories [9, 3, 15].

Consider a game with m strategies and let p_i be the frequency of strategy i in the population. At any given time the state of the population is given by $\mathbf{p} \in \mathbf{S_m}$. If p_i is a differential function of time, then the evolution of strategies in the population can be expressed using *replicator equations* [8]. (Differentiability assumes the population

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is infinitely large.) Each replicator equation is a 1st-order differential equation. Under replication dynamics individuals do not change strategies via mutation nor by some contrived procedure such as a Moran process [10]. Instead, strategies change frequency following Darwinian theory—i.e., reproductive success is directly proportional to fitness. Individuals with above average fitness grow in the population over time while those with below average fitness die out.

Usually the N player games studied this far only have a small number of strategies (typically $m \leq 4$). Most of these games study cooperation in populations and try to discover human characteristics that promote cooperation. These games model social dilemmas where mutual cooperation is the best outcome for the group, but individual self-interest always pays better, leading to the undesirable outcome where ultimately everyone defects. Empirical evidence from these N player games suggest that several mechanisms such as reciprocity and kin-selection promote cooperation in human populations [11].

One aspect of human interaction that has been extensively investigated in the past is trust and the role it plays in society. Schmueli *et.al.* [14] maintain that the concept of trust is pervasive in social relationships and it has a great impact on social persuasion and behavioral change. Their experiment revealed that trust was significantly more effective than the closeness of ties in determining the amount of behavior change, with respect to individual fitness. High levels of trust have shown to impose social controls in political and economic institutions thereby increasing accountability, productivity and effectiveness [13].

Nevertheless, evolutionary game theoretical studies on trust are lacking and those that have been conducted were limited to 2 players. (See [12] for a notable exception.) Recently this situation changed when Abbass et. al [1] introduced a game specifically designed to investigate the role of trust in human populations. Players in this game make two choices in advance: whether to be trustworthy or not and whether to be an investor or be a trustee. Each investor contributes an amount tv. A trustworthy trustee returns an amount $R1 > 1$ to the investor (and keeps an equal amount for herself) whereas an untrustworthy trustee keeps the contribution and returns nothing. The game is designed as a social dilemma. Replicator dynamics indicate that the inevitable outcome is when the population converges to state with a mixture of trustworthy and untrustworthy players and no investors.

Replicators equations provide valuable insights into how strategies evolve in a population. Their limitation is the assumption of an infinite population. Nature does not produce infinite populations. Indeed, human populations are always finite for a variety of reasons such as geographical isolation or cultural restrictions. More importantly, it has been conclusively shown that, when comparing infinite population dynamics and finite population dynamics, the latter have qualitatively different results [4, 6, 5]. This issue is important because the trust game results reported in [1] were obtained using an infinite population model.

In this paper, we extend our previous work by studying finite population models using a discrete form of replicator equations and report the findings. Our results indicate the finite population dynamics are remarkably similar to those found in the infinite population. However, the discrete replicator equations do require quantization and

Table 1. Utility matrix for a N-player trust game.

	Player in the k_1 population	**Player in the k_2 population**	**Player in the k_3 population**
Pay	tv	$R1 \cdot tv \cdot \frac{k_1}{k_2+k_3}$	0
Receive	$R1 \cdot tv \cdot \frac{k_2}{k_2+k_3}$	$2 \cdot R1 \cdot tv \cdot \frac{k_1}{k_2+k_3}$	$R2 \cdot tv \cdot \frac{k_1}{k_2+k_3}$
Net Wealth	$tv \cdot (R1 \cdot \frac{k_2}{k_2+k_3} - 1)$	$R1 \cdot tv \cdot \frac{k_1}{k_2+k_3}$	$R2 \cdot tv \cdot \frac{k_1}{k_2+k_3}$

quantization effects introduce additional fixed points not found in the infinite population models. Surprisingly, these fixed points appear and disappear as a function of the population size. We provide an analysis of this phenomenon.

This paper is organized as follows. In Section 2 the trust game is formally defined and an overview of the infinite population replicator dynamics is given. Section 3 develops the replicator equations for the trust game with finite populations. Section 4 analyzes the finite population results. Finally, Section 5 summarizes our findings and discusses future work.

2 Background

This section gives a formal definition of the N player trust game and a brief overview of the infinite population replicator dynamics. See [1] for more detailed information.

2.1 The N Player Trust Game

Assume N players. Each player makes two decisions in advance: (1) whether or not to be trustworthy, and (2) whether to be an investor or a trustee. Let k_1 be the number of investors, k_2 the number of trustworthy trustees and k_3 the number of untrustworthy trustees. The obvious restriction is $\sum_i k_i = N$.

An investor player pays tv to the trustee, where $tv > 0$ denotes the trusted value. The dynamics of the game does not depend on the value of tv. However, we maintain tv to allow flexibility in adopting the game to different contexts. With k_1 governed players, the total money contributed is $(k_1 \cdot tv)$. Each trustworthy trustee returns to an investor a multiplier of $R1$ of what was received and keeps the same amount for herself, with $R1 > 1$. An untrustworthy trustee returns nothing but instead keeps for herself a multiplier of $R2$ of what was received, where $R1 < R2 < 2R1$. The payoff matrix for this game can then be represented as shown in Table 1 with the following constraints:

$$1 < R1 < R2 < 2R1$$

$$N = k_1 + k_2 + k_3$$

2.2 Infinite Population Replication Dynamics

The evolutionary behavior of a population playing the trust game can be studied using replicator dynamics. Let y_i be the frequency of players using strategy i in an infinitely large population with $\sum_i y_i = 1$. Then the time evolution of y_i is given by the differential equation

$$\dot{y}_i = y_i \cdot \left(f_i - \hat{f}\right) \tag{1}$$

where f_i is the expected fitness of an individual playing strategy i at time t and $\hat{f}$ is the mean population fitness. Here, fitness and net wealth are equivalent. The number of copies of a strategy increases if $f_i > \hat{f}$ and decreases if $f_i < \hat{f}$. We can calculate $\hat{f}$ as follows

$$\hat{f} = \frac{y_1 \cdot y_2 \cdot tv\,(2 \cdot R1 - 1) + y_1 \cdot y_3 \cdot tv \cdot (R2 - 1)}{(y_2 + y_3)}$$

The three replicator equations are

$$\dot{y}_1 \;=\; \frac{y_1^2 \cdot tv}{1 - y_1}\,(y_2\,(1 - 2 \cdot R1) + y_3 \cdot (1 - R2)) \;+\; \frac{y_1 \cdot tv}{1 - y_1}\,(y_2\,(R1 - 1) - y_3)$$

$$\dot{y}_2 = \frac{y_1 \cdot y_2 \cdot tv}{1 - y_1} \cdot (y_2\,(1 - 2 \cdot R1) + y_3\,(1 - R2) + R1)$$

$$\dot{y}_3 = \frac{y_1 \cdot y_3 \cdot tv}{1 - y_1} \cdot (y_2\,(1 - 2 \cdot R1) + y_3\,(1 - R2) + R2)$$

Figure 1 shows the population evolution for various initial player distributions. Figure 2 shows the effect of different $R1$ and $R2$ values (but with $R1 < R2$). The replicator equations predict a rapid growth of untrustworthiness in the population, leading to the eventual extinction of investors. However, a fraction of the population always remains trustworthy, even in the absence of investors. This predicted steady-state outcome is independent of the initial player distribution, but the ratio of trustworthy to untrustworthy players in the final population is dependent on the $R1$ and $R2$ values.

3 Finite Population Replicator Dynamics

The population dynamics depicted in Figure 1 and predicted by (1) apply only to infinite size populations. It has been conclusively shown that finite population dynamics can be qualitatively different from those of infinite populations [4, 6, 5]. It is therefore important to see if and, if so, how the dynamics of the trust game change for finite populations. Replicator equations can still be used to predict how a population evolves although, as will be shown shortly, the equation format is different.

Let N be the (finite) population size and $k_i,\ i = \{1, 2, 3\}$ be the number of players choosing strategy i. The frequency of players choosing strategy i at time t is $p_i^t = k_i/N$.

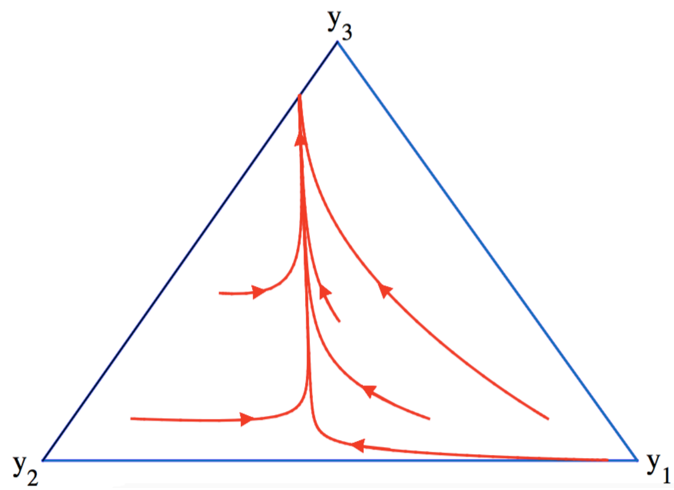

Fig. 1. A 2-simplex showing the time evolution for a game with $R1 = 6$, $R2 = 8$, $tv = 10$, and different initial distributions of y_1, y_2 and y_3. (Reproduced from [1])

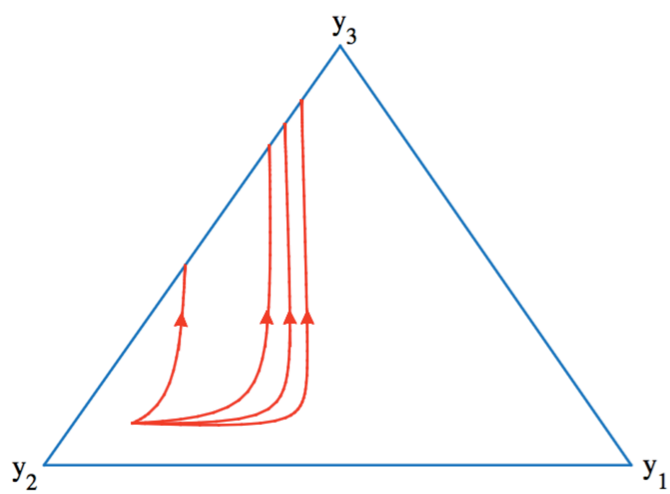

Fig. 2. A 2-simplex showing the time evolution for a game with $tv = 10$ and different $R1$ and $R2$ values ($R1 < R2$). Values increase from left to right with $R1 = 1.5$, $R2 = 2.9$ for the far left trajectory to $R1 = 6$, $R2 = 8$ for the far right trajectory. Initial distribution is $y_1(0) = 0.1$, $y_2(0) = 0.8$, and $y_3(0) = 0.1$.(Reproduced from [1])

With finite populations, the discrete replicator equations are now expressed as a set of first order difference equations

$$p_i^{t+1} = p_i^t F_i^t \tag{2}$$

where $F_i^t = f_i \Big/ \hat{f}^t$ with $\hat{f}^t$ the mean fitness at time t. $F_i > 1$ means the proportion of strategy i in the population grows, $F_i < 1$ means it shrinks and $F_i = 1$ it is at a fixed point. Unfortunately, the discrete form of the replicator equations introduces a couple of problems not found in the infinite population case.

The first problem is with the definition of fitness. Fitness is equated to net wealth in both the infinite population case and the finite population case. Substituting $p_i^t = k_i/N$ into the net wealth equations and simplifying yields the following finite population fitness equations

$$\begin{aligned} f_1^t &= tv \cdot \left(R1 \cdot \tfrac{p_2^t}{p_2^t + p_3^t} - 1\right) \\ f_2^t &= tv \cdot R1 \cdot \tfrac{p_1^t}{p_2^t + p_3^t} \\ f_3^t &= tv \cdot R2 \cdot \tfrac{p_1^t}{p_2^t + p_3^t} \end{aligned} \tag{3}$$

The problem is f_1 won't be positive for all strategy frequencies. Unlike with differential equations, negative fitness values are not permitted in discrete replicator equations because this makes $p_i^{t+1} < 0$. Moreover, $\hat{f}^t$ cannot equal zero. We therefore slightly modified the fitness values as shown below.

$$\begin{aligned} f_1 &= \begin{cases} 0 < \epsilon \ll 1 & \text{if } \frac{k_2}{k_2+k_3} \leq \frac{1}{R1} \\ tv \cdot \left(R1 \cdot \frac{k_2}{k_2+k_3} - 1\right) & \text{otherwise} \end{cases} \\ f_2 &= tv \cdot R1 \cdot \tfrac{k_1}{k_2+k_3} \\ f_3 &= tv \cdot R2 \cdot \tfrac{k_1}{k_2+k_3} \end{aligned} \tag{4}$$

The second problem involves trajectories in the 2-simplex. In the finite population case each $p_i^t = k_i/N$. This means there are only a finite set of feasible points in the 2-simplex (see Figure 3). Any trajectory must therefore consist of straight line segments between pairs of feasible points.
Clearly, the right-hand side must be an integer. This means only a finite set of points in the simplex can be visited—i.e., points where $p_i^t = k_i/N$. These points for $N = 20$ are shown in Figure 3.

The p_i values have to be quantized to make sure only integer values for k_i^{t+1} are produced. The quantization method described in [2] is used here. The algorithm below returns k_i' which is the new number of players choosing the i-th strategy.

1. Compute
$$k_i' = \left\lfloor N p_i + \tfrac{1}{2} \right\rfloor \quad , \quad N' = \sum_i k_i'$$
2. Let $d = N' - N$. If $d = 0$, then go to step 4. Otherwise, compute the errors $\delta_i = k_i' - N p_i$.
3. If $d > 0$, decrement the d k_i''s with the largest δ_i values. If $d < 0$, increment the $|d|$ k_i''s with the smallest δ_i values.

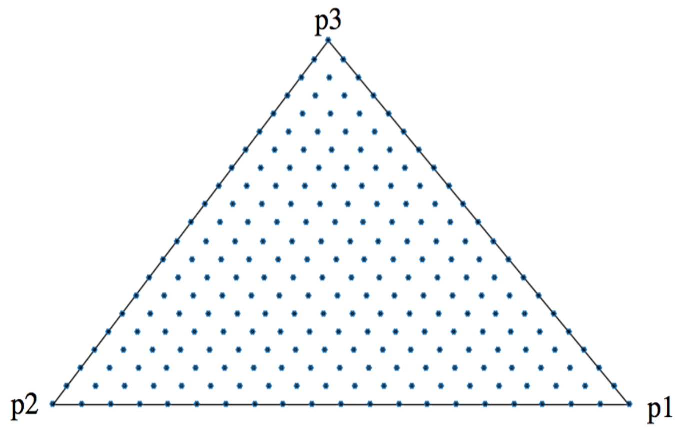

Fig. 3. A 2-simplex for a finite population with $N = 20$. Only the points shown represent an integer number of strategies in the population. Any trajectory must move between these points.

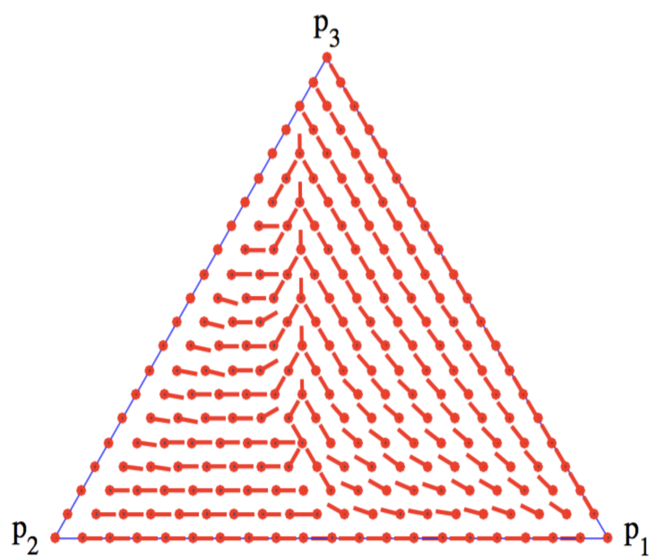

Fig. 4. A velocity plot for a finite population with $N = 20, R1 = 6, R2 = 8$ and $tv = 10$. (*c.f.* Figure 1)

4. Return $[k'_1 \quad k'_2 \quad k'_3]$ and exit.

Figure 4 shows a velocity plot for a finite population with $N = 20, R1 = 6, R2 = 8$ and $tv = 10$. The vectors are shown as unit vectors because the direction is of importance here and not the magnitude. These finite population replicator dynamics are the analog of the infinite population replicator dynamics shown in Figure 1. Notice the finite population dynamics are remarkably similar including the presence of an attractor.

4 Discussion

The finite population trust game has a Nash equilibrium at $k_3 = N$ and a Pareto optimal distribution at $k_1 = N - 1, k_2 = 1$. (See [1] for proofs.)

Many of the fixed points in the finite population are the same as those in the infinite population. For example, in the infinite population the three 2-simplex corners and every point on the $p_2 - p_3$ line is a fixed point. Similarly in the finite population model the three 2-simplex corners are fixed points but only a finite number of points on the $p_2 - p_3$ line are fixed points—i.e., the $N + 1$ points where p_i is a rational number. The infinite population model also has a fixed point at

$$y_1 = \frac{R1 - 1}{2 \times R1 - 1}$$

$$y_2 = \frac{R1}{2 \times R1 - 1}$$

$$y_3 = 0$$

With R1 = 6 in our example, the fixed point is, $\mathbf{p} = [5/11\ 6/11\ 0/11]$. In the finite model this fixed point varies (due to quantization) but it is the rational number closest to $\mathbf{p}$.

Figure 5 shows a magnified view of a portion of the $p_1 - p_2$ line. Notice there are two fixed points that do not appear in the infinite population mode. Consider the fixed point at $[p_1\ p_2\ p_3] = [0.450\ 0.550\ 0.000]$ (equivalently, $[k_1\ k_2\ k_3] = [9\ 11\ 0]$). That population mixture yields fitness values of $f_1 = 50, f_2 = 49.09, f_3 = 0.0$ and a mean population fitness of $\hat{f} = 49.499$. The discrete replicator equations predict no change in the population mixture. It is also worth mentioning the fixed point on the $p_1 - p_2$ line at $[p_1\ p_2\ p_3] = [0.450\ 0.550\ 0.000]$ matches well to the fixed point in the infinite population at $[p_1\ p_2\ p_3] = [0.455\ 0.545\ 0.000]$.

To understand why the finite population has fixed points which are not present in infinite populations it is important to understand how quantization actually works. Quantization is a form of data compression. It maps an entire range of real numbers into a single value, which subsequently represents any real number in the range that was mapped.

For the trust game quantization must map real numbers into integers. To see why this is necessary substitute $p_i^t = k_i^t/N$ into the discrete replicator equation. After multiplying both sides of the equation by N

$$k_i^{t+1} = k_i^t F_i^t \tag{5}$$

Clearly the left-hand side must be an integer but the right-hand side typically won't be because $F_i^t = f_i^t/\hat{f}^t$ is a real number. The quantization process described previously was specifically picked because it maps a real number into an integer. However, there is no guarantee quantizing three frequencies that sum to 1.0 will produce three integers

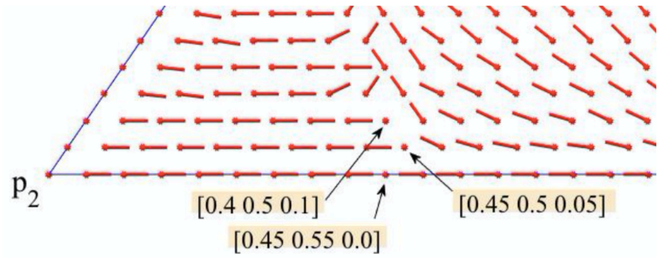

Fig. 5. A magnified portion of the 2-simplex for $N = 20$. The distribution of strategies is shown for the three fixed points. The fixed point with distribution [0.45 0.55 0.0] corresponds to a similar fixed in the infinite population module. The other two fixed points are a consequence of quantization (see text).

that sum to N—unless a repair mechanism is incorporated into the quantization process to enforce this constraint.

Each iteration of the discrete replicator equation updates the number of strategies in the population. Thus update is a mapping from $I \to I$. Unfortunately the right-hand side of (2) is rarely an integer. Quantization will produce an integer right-hand side of the replicator equation but this process introduces some fixed points not present in the infinite population model. To understand how these fixed points arise it is necessary to take a more detailed look at the quantization process.

Step 1 of the quantization process computes the new number of the i-th strategy:

$$\begin{aligned} k'_i &= \left\lfloor N p_i^{t+1} + \tfrac{1}{2} \right\rfloor \\ &= \left\lfloor N \tfrac{k_i}{N} F_i^t + \tfrac{1}{2} \right\rfloor \\ &= \left\lfloor k_i F_i^t + \tfrac{1}{2} \right\rfloor \end{aligned} \tag{6}$$

where the integer floor is necessary to make sure k'_i is an integer. It is easy to show $k_i \to k'_i$ as follows

$$k'_i = \begin{cases} < k_i & \text{if } F_i^t < 1 - 1/2k_i \\ > k_i & \text{if } F_i^t > 1 + 1/2k_i \\ = k_i & \text{otherwise} \end{cases} \tag{7}$$

The new sum $\sum_i k'_i = N'$ is calculated and then compared with N. Obviously no adjustment is required if $N' = N$. However, if $N' \neq N$ then some k'_i values must be incremented (if $N' < N$) or decremented (if $N' > N$). This adjustment is done in steps 2 through 4 of the quantization process. Which ones get incremented or decremented depends on the δ_i error values: those with the largest errors are decremented and those with the smallest errors get incremented. Thus, the only role of the δ_i values is to identify which k'_i's must be adjusted to make $N' = N$.

Now consider the upper fixed point highlighted in Figure 5 where $N = 20$. The population mixture is $[p_1\ p_2\ p_3] = [0.4\ 0.5\ 0.1]$ at that simplex point. The corresponding fitness values are $[f_1\ f_2\ f_3] = [40\ 40\ 53.33]$ and the mean fitness is $\hat{f} = 41.33$. Consequently $F_1^t = F_2^t = 0.967$ and $F_3^t = 1.29$. Thus,

1. $k_1' = k_1$ because $0.967 \not< 1 - \frac{1}{16}$ and $0.967 \not> 1 + \frac{1}{16}$.
2. $k_2' = k_2$ because $0.967 \not< 1 - \frac{1}{20}$ and $0.967 \not> 1 + \frac{1}{20}$.
3. $k_3' > k_3$ because $1.29 > 1 + \frac{1}{4}$. (Note: $k_3 = 2$ and $\left\lfloor 1.29 \cdot 2 + \frac{1}{2} \right\rfloor = \lfloor 3.08 \rfloor$)

Readjustment is necessary because $\sum_i k_i' = 21 > N$. Step 3 of the quantization algorithm implements the repair mechanism. Specifically, in this case $d = +1$ and δ_3 is larger than δ_1 or δ_2. Thus k_3 is decremented once, which makes $k_3' = k_3$. Now $\sum_i k_i = N$ and fixed point is created since none of the k_is changed. A similar analysis can be done for the fixed point with the distribution $[0.45\ 0.5\ .05]$.

The fixed points in the interior of the 2-simplex caused by quantization will change as N increases and they completely disappear as $N \to \infty$. To investigate this phenomenon a simulation was run with $N = 40$. Figure 6 shows a magnified portion of the 2-simplex. The fixed point with distribution $[0.45\ 0.55\ 0.0]$ remains and will never disappear as N increases. Notice a new fixed point appeared at distribution $[0.45\ 0.525\ .025]$. More importantly, the fixed point at distribution $[0.4\ 0.5\ 0.1]$, which was a fixed point when $N = 20$ is no longer a fixed point when $N = 40$. An analysis conducted as done above will explain why. The same fitness values exist but now the new strategy numbers are as follows

1. $k_1' < k_1$ because $0.967 < 1 - \frac{1}{32}$. ($k_1 = 16$; $k_1' = 15$)
2. $k_2' < k_2$ because $0.967 < 1 - \frac{1}{40}$. ($k_2 = 20$; $k_2' = 19$)
3. $k_3' > k_3$ because $1.29 > 1 + \frac{1}{8}$. ($k_3 = 4$; $k_3' = 5$)

Readjustment is necessary because $\sum_i k_i' = 39 < N$. From step 2 of the quantization algorithm $\delta_1 = -0.472, \delta_2 = -0.34, \delta_3 = -0.16$ and $d = -1$. δ_1 is the smallest so k_1' is increased from 15 to 16 making the number of strategies total to N. The simplex point with strategy distribution $[0.4\ 0.5\ 0.1]$ is no longer a fixed point when $N = 40$ because $k_2' \neq k_2$ and $k_3' \neq k_3$.

One area where the finite population replicator dynamics differs markedly from the infinite population dynamics is the region in the 2-simplex near the p_2 vertex (see Figure 7). The presence of an attractor is obvious but, unlike the infinite population case, the fixed points on the $p_2 - p_3$ axis are not unique (*c.f.*, Figure 2). Most likely this is another effect of quantization.

5 Summary

In this paper we have extended our previous work on the N player trust game by studying finite population effects. The discrete replicator equations impose certain restrictions including the necessity of quantization. Quantization introduces fixed points in the interior of the 2-simplex but these disappear (and other may take their place) as N

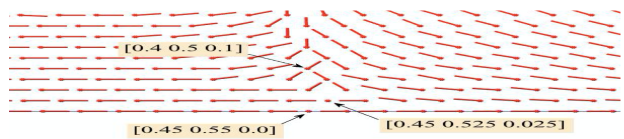

Fig. 6. A magnified portion of the 2-simplex for $N = 40$. The distribution of strategies is shown for the three fixed points. The fixed point with distribution [0.45 0.55 0.0] corresponds to a similar fixed in the infinite population module. The other fixed point is a consequence of quantization (see text).

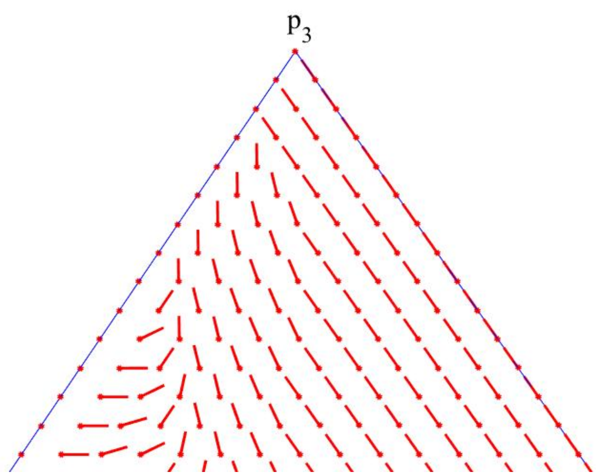

Fig. 7. A magnified portion of the 2-simplex for $N = 40$. (*c.f.*, Figure 1)

varies. Nevertheless, the finite population evolution is qualitatively similar to the infinite population. This research extended previous research on computational measurement of trust by using game theory in both finite and infinite populations.

The replicator equations describe strategy evolution based on Darwinian principles—i.e., fitness based evolution. In particular no mutation is permitted. It will be interesting to see if trust persists when individuals are allowed to modify their strategy. Trust is the foundation of all human interactions regardless of who is involved or the duration of the encounter. This suggests emotions may play a role in whether or not individuals are seen as trustworthy and, if so, for how long. Greenwood [7], has previously shown emotions such as guilt can affect cooperation levels in social dilemmas. In our future work we intend to see how emotions may affect trust levels in social dilemmas.

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